and making the assumption that the latter has only one term, for which n_0^2 is large compared with n^2, we have

$$a = C/n_0^2, \qquad b = \lambda_0^2.$$

If $b = p + qa$ for all the inert gases it would follow that the square of the wave-length of the free vibration dominating the dispersion is equal to a constant plus a term proportional to the refractivity for infinite wave-length.

We have great pleasure in expressing our thanks to Sir William Ramsay and Prof. R. B. Moore for the loan of the gases; to Prof. Trouton, Prof. Porter, and the staff of the Physical Laboratory of University College, London, for much assistance and advice; and to the Royal Society for a grant in aid of these experiments.

Note on Tidal Bores.

By Lord Rayleigh, O.M., Pres. R.S.

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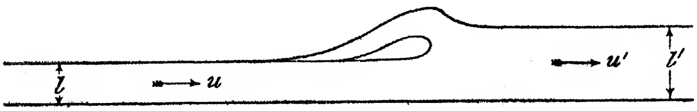
It was shown long ago by Airy that when waves advance over shallow water of depth originally uniform, the crests tend to gain upon the hollows,* so that the anterior slopes become steeper and steeper. Ultimately, if the conditions are favourable, there formed what is be may called a *bore*. Ordinary breakers upon a shelving beach are of this character, but the name is usually reserved for tidal bores advancing up rivers or estuaries. Interesting descriptions of some of these are given in Sir G. Darwin's 'Tides' (Murray, 1898).

Although the real bore advances up the channel, we may for theoretical purposes "reduce it to rest" by superposing an equal and opposite motion upon the whole water system. We have then merely to investigate the transition from a relatively rapid and shallow stream of depth l and velocity u to a deeper and slower stream of depth l' and velocity u' (fig. 1). The

Fig. 1.



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* See also 'Scientific Papers,' vol. 1, p. 253, 1899.

places where these velocities and depths are reckoned are supposed to be situated on the two sides of the bore and at such distances from it that the motions are there sensibly uniform. The problem being taken as in two dimensions, two relations may at once be formulated connecting the depths and velocities. By conservation of matter (" continuity ") we have

$$lu = l'u'. \tag{1}$$

And since the mean pressures at the two sections are $\frac{1}{2}gl$, $\frac{1}{2}gl'$, the equation of momentum is

$$lu(u-u') = \tfrac{1}{2}g(l'^2-l^2); \tag{2}$$

whence
$$u^2 = \tfrac{1}{2}g(l+l').l'/l, \qquad u'^2 = \tfrac{1}{2}g(l+l').l/l'. \tag{3}$$

The loss of energy per unit time at the bore is thus

$$lu(\tfrac{1}{2}u^2+\tfrac{1}{2}gl)-lu(\tfrac{1}{2}u'^2+\tfrac{1}{2}gl') = lu.g(l'-l)\frac{l^2+l'^2}{4ll'}. \tag{4}$$

That there should be a loss of energy constitutes no difficulty, at least in the presence of viscosity; but the impossibility of a gain of energy shows that the motions here contemplated cannot be reversed.

In order to recur to the natural condition of things where the shallow water is at rest, we have to superpose the velocity u taken negatively upon the above motion. The velocity of the bore is then u and that of the stream above the bore $u - u'$. If l is relatively small, u is much greater than u'.

The reasoning just used is very similar to that applied by Stokes* and by Riemann† to sound waves of expansion moving in one dimension. The matter is discussed in 'Theory of Sound,' § 253, where it is shown that the discontinuous solution, obtained from the principles of conservation of mass and momentum, violates the condition of energy. When this was pointed out to Stokes by Kelvin and later by myself,‡ he abandoned his solution, which is, however, maintained by a competent German authority.§ It is clear, at least, that when the motion is such as to involve a gain of energy, the solution cannot be admitted. The opposite case stands upon a different footing, and we may, perhaps, imagine the redundant mechanical energy to be got rid of somehow at the surface of discontinuity. Even then we should have to face the complication entailed by the development of heat. In the present case of liquid, the heat is of little consequence, and since the motion is not entirely in one dimension, we escape the necessity of dealing with a single plane of discontinuity.

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* 'Phil. Mag.,' vol. 33, p. 349, 1848.

† 'Göttingen Abh.,' vol. 8, 1860.

‡ Stokes, 'Math. and Phys. Papers,' vol. 2, p. 55.

§ Private correspondence.